Note: In this problem set, expressions in green cells match corresponding expressions in the text answers.

```
Clear["Global`*"]
```

3. Which are the “bottleneck” edges by which the flow in example 1 is actually limited? Hence which capacities could be decreased without decreasing the maximum flow?

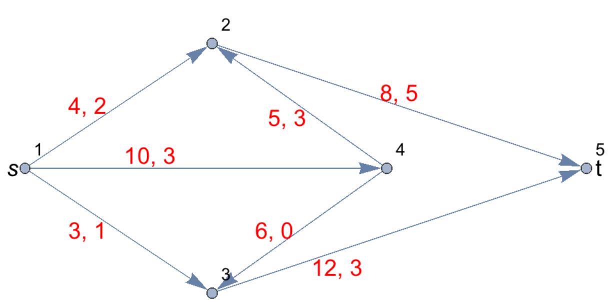
5. How does Ford-Fulkerson prevent the formation of cycles?

6 - 9 Maximum flow
Find the maximum flow by Ford-Fulkerson:

7. In problem 15, section 23.6.

I’m not using Ford-Fulkerson, just the standard Mathematica command. Incidentally I ran across a statement on line that F-F incorporates a repetitive or redundant structure, and that both Dinic and Edmonds-Karp are improved versions of the algorithm.

```
g15 = Graph[
  {2 ⟷ 5, 4 ⟷ 2, 4 ⟷ 3, 1 ⟷ 2, 3 ⟷ 5, 1 ⟷ 3, 1 ⟷ 4}, VertexLabels → "Name",
  VertexCoordinates -> {{0, 0}, {3, -1}, {1.4, -1}, {0, -2}, {-1.5, -1}},
  EdgeCapacity → {8, 5, 6, 4, 1, 3, 10}, EdgeWeight → {5, 3, 0, 2, 1, 1, 3},
  Epilog → {{Text[Style["s", Medium], {-1.6, -1}]},
    {Red, Text[Style["3, 1", Medium], {-1, -1.5}]},
    {Red, Text[Style["10, 3", Medium], {-0.5, -0.9}]},
    {Red, Text[Style["4, 2", Medium], {-1, -0.5}]},
    {Red, Text[Style["6, 0", Medium], {0.5, -1.5}]},
    {Text[Style["t", Medium], {3.1, -1}]},
    {Red, Text[Style["5, 3", Medium], {0.6, -0.6}]},
    {Red, Text[Style["8, 5", Medium], {1.5, -0.4}]},
    {Red, Text[Style["12, 3", Medium], {1, -1.8}]}},
  ImageSize → 350, ImagePadding → 20]
```



```
gdc15 = FindMaximumFlow[Graph[
   {2 •→ 5, 4 •→ 2, 4 •→ 3, 1 •→ 2, 3 •→ 5, 1 •→ 3, 1 •→ 4}, VertexLabels → "Name",
   VertexCoordinates -> {{0, 0}, {3, -1}, {1.4, -1}, {0, -2}, {-1.5, -1}},
   EdgeCapacity → {8, 5, 6, 4, 1, 3, 10},
   EdgeWeight → {5, 3, 0, 2, 1, 1, 3}], 1, 5, "OptimumFlowData"]
```

```
OptimumFlowData[ [+] Flowvalue 9 ]
```

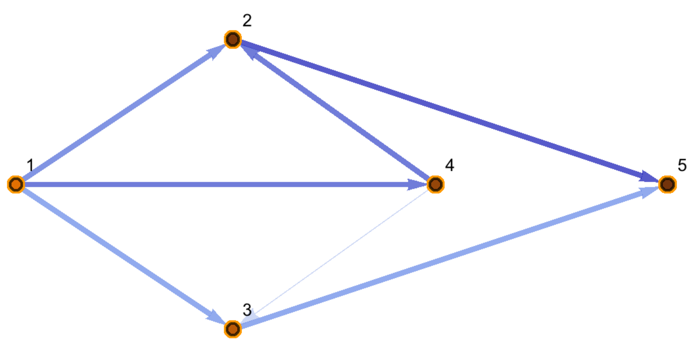
The green cell above contains the value in agreement with the text answer, for the flow. To inventory the contributions and edge structure I go on. The next cell shows what was made available by executing the **OptimumFlowData** option.

```
gdc15["Properties"]
```

```
{CostValue, EdgeList, FlowGraph, FlowMatrix,
 FlowTable, FlowValue, Properties, ResidualGraph, VertexList}
```

The flow graph shows all the original edges, but the grid shows that 4•→3 is not nonzero.

```
gdc15["FlowGraph"]
```

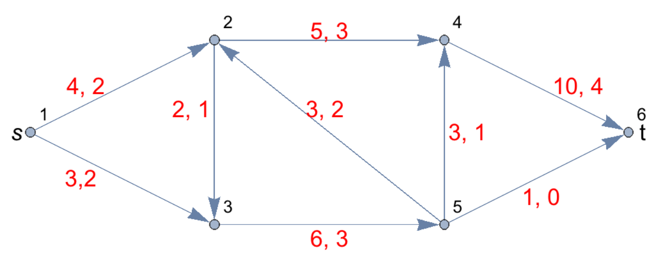



```
Grid[{#, gdc15[#]} & /@ gdc15["EdgeList"], Frame → All]
```

2 •→ 5	8
4 •→ 2	5
3 •→ 5	1
1 •→ 2	3
1 •→ 3	1
1 •→ 4	5

9. Problem represented by a diagram.

```
g9 = Graph[{1 ↦ 2, 2 ↦ 4, 4 ↦ 6, 3 ↦ 5, 5 ↦ 6, 1 ↦ 3, 5 ↦ 2, 5 ↦ 4, 2 ↦ 3},
  VertexLabels → "Name", VertexCoordinates ->
   {{-2, -1}, {0, 0}, {2.5, 0}, {4.5, -1}, {0, -2}, {2.5, -2}},
  EdgeCapacity → {4, 5, 10, 6, 1, 3, 3, 3, 2},
  EdgeWeight → {2, 3, 4, 3, 0, 2, 2, 1, 1},
  Epilog → {{Text[Style["s", Medium], {-2.15, -1}]},
    {Red, Text[Style["3,2", Medium], {-1.45, -1.5}]},
    {Red, Text[Style["3, 1", Medium], {2.75, -1}]},
    {Red, Text[Style["4, 2", Medium], {-1.4, -0.5}]},
    {Red, Text[Style["1, 0", Medium], {3.55, -1.7}]},
    {Text[Style["t", Medium], {4.65, -1}]},
    {Red, Text[Style["3, 2", Medium], {1.2, -0.75}]},
    {Red, Text[Style["5, 3", Medium], {1.25, 0.1}]},
    {Red, Text[Style["6, 3", Medium], {1.25, -2.15}]},
    {Red, Text[Style["10, 4", Medium], {3.95, -0.5}]},
    {Red, Text[Style["2, 1", Medium], {-0.25, -0.75}]}},
  ImageSize → 400, ImagePadding → 35]
```

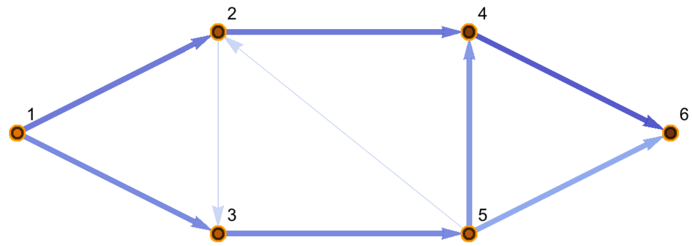



```
gdc9 = FindMaximumFlow[
  Graph[{1 ↦ 2, 2 ↦ 4, 4 ↦ 6, 3 ↦ 5, 5 ↦ 6, 1 ↦ 3, 5 ↦ 2, 5 ↦ 4, 2 ↦ 3},
   VertexLabels → "Name", VertexCoordinates ->
    {{-2, -1}, {0, 0}, {2.5, 0}, {4.5, -1}, {0, -2}, {2.5, -2}},
   EdgeCapacity → {4, 5, 10, 6, 1, 3, 3, 3, 2}], 1, 6, "OptimumFlowData"]
```

```
OptimumFlowData[ + Flowvalue 7 ]
```

The cell above contains the calculated flow value which agrees with the text answer. In the diagram below, two edges do not make contributions, but are shown, I guess, for completeness.

```
gdc9["FlowGraph"]
```



The grid shows how the flow reaches t.

```
Grid[{#, gdc9[#]} & /@ gdc9["EdgeList"], Frame → All]
```

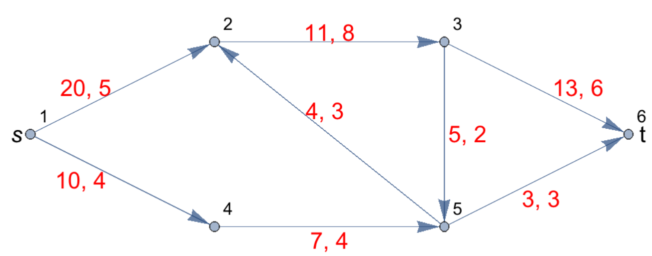
1 ↦ 2	4
1 ↦ 3	3
2 ↦ 4	4
4 ↦ 6	6
3 ↦ 5	3
5 ↦ 6	1
5 ↦ 4	2

15. Find a minimum cut set in figure 500 and its capacity.

```
Get["IGraphM`"]
```

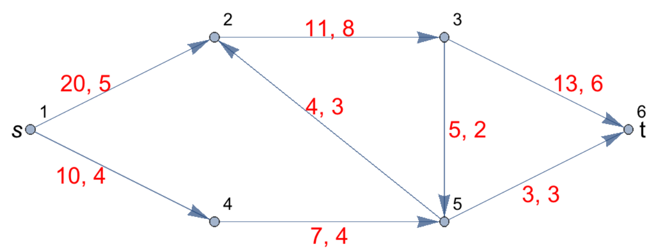
```
IGraph/M 0.3.110 (April 22, 2019)

Evaluate IGDocumentation[] to get started.
```

```
g15 = Graph[{1 ⧴ 2, 2 ⧴ 3, 3 ⧴ 6, 4 ⧴ 5, 5 ⧴ 6, 1 ⧴ 4, 5 ⧴ 2, 3 ⧴ 5},
  VertexLabels → "Name", VertexCoordinates ->
   {{-2, -1}, {0, 0}, {2.5, 0}, {4.5, -1}, {0, -2}, {2.5, -2}},
  EdgeCapacity → {20, 11, 13, 7, 3, 10, 4, 5},
  EdgeWeight → {5, 8, 6, 4, 3, 4, 3, 2},
  Epilog → {{Text[Style["s", Medium], {-2.15, -1}]},
    {Red, Text[Style["10, 4", Medium], {-1.45, -1.5}]},
    {Red, Text[Style["5, 2", Medium], {2.75, -1}]},
    {Red, Text[Style["20, 5", Medium], {-1.4, -0.5}]},
    {Red, Text[Style["3, 3", Medium], {3.55, -1.7}]},
    {Text[Style["t", Medium], {4.65, -1}]},
    {Red, Text[Style["4, 3", Medium], {1.2, -0.75}]},
    {Red, Text[Style["11, 8", Medium], {1.25, 0.1}]},
    {Red, Text[Style["7, 4", Medium], {1.25, -2.15}]},
    {Red, Text[Style["13, 6", Medium], {3.95, -0.5}]}},
  ImageSize → 400, ImagePadding → 35]
```



First I will try out the IG functions. It is necessary to move the **`EdgeCapacity`** properties into the **`EdgeWeight`** block, because edge weights are what IG will look for in calculating flows.

```
g15a = Graph[{1 ↔ 2, 2 ↔ 3, 3 ↔ 6, 4 ↔ 5, 5 ↔ 6, 1 ↔ 4, 5 ↔ 2, 3 ↔ 5},
  VertexLabels → "Name", VertexCoordinates ->
   {{-2, -1}, {0, 0}, {2.5, 0}, {4.5, -1}, {0, -2}, {2.5, -2}},
  EdgeCapacity → {20, 11, 13, 7, 3, 10, 4, 5},
  EdgeWeight → {20, 11, 13, 7, 3, 10, 4, 5},
  Epilog → {{Text[Style["s", Medium], {-2.15, -1}]},
    {Red, Text[Style["10, 4", Medium], {-1.45, -1.5}]},
    {Red, Text[Style["5, 2", Medium], {2.75, -1}]},
    {Red, Text[Style["20, 5", Medium], {-1.4, -0.5}]},
    {Red, Text[Style["3, 3", Medium], {3.55, -1.7}]},
    {Text[Style["t", Medium], {4.65, -1}]},
    {Red, Text[Style["4, 3", Medium], {1.2, -0.75}]},
    {Red, Text[Style["11, 8", Medium], {1.25, 0.1}]},
    {Red, Text[Style["7, 4", Medium], {1.25, -2.15}]},
    {Red, Text[Style["13, 6", Medium], {3.95, -0.5}]}},
  ImageSize → 400, ImagePadding → 35]
```



Then I can call the functions. The IG functions make the same cut as the text answer, and come up with the same flow.

```
IGMinimumCut[g15a, 1, 6]
```

```
{2 ↔ 3, 5 ↔ 6}
```

```
IGMinimumCutValue[g15a, 1, 6]
```

```
14.
```

I still want to take a look at using the Mathematica graph toolset. For this I modify the graph to simply drop the edge weights, since Mathematica will not look at them anyway.

```
Clear["Global`*"]
```

```
g155 = Graph[{1 ↔ 2, 2 ↔ 3, 3 ↔ 6, 4 ↔ 5, 5 ↔ 6, 1 ↔ 4, 5 ↔ 2, 3 ↔ 5},
   VertexLabels → "Name", VertexCoordinates ->
    {{-2, -1}, {0, 0}, {2.5, 0}, {4.5, -1}, {0, -2}, {2.5, -2}},
   EdgeCapacity → {20, 11, 13, 7, 3, 10, 4, 5},
   ImageSize → 400, ImagePadding → 35];
```

I use **FindMaximumFlow** with **OptimumFlowData** so I can get the full menu of available properties. Mathematica reports the maximum flow.

```
g156 = FindMaximumFlow[
  Graph[{1 ↦ 2, 2 ↦ 3, 3 ↦ 6, 4 ↦ 5, 5 ↦ 6, 1 ↦ 4, 5 ↦ 2, 3 ↦ 5},
   VertexLabels → "Name", VertexCoordinates ->
    {{-2, -1}, {0, 0}, {2.5, 0}, {4.5, -1}, {0, -2}, {2.5, -2}},
   EdgeCapacity → {20, 11, 13, 7, 3, 10, 4, 5}], 1, 6, "OptimumFlowData"]
```

```
OptimumFlowData[ Flowvalue 14 ]
```

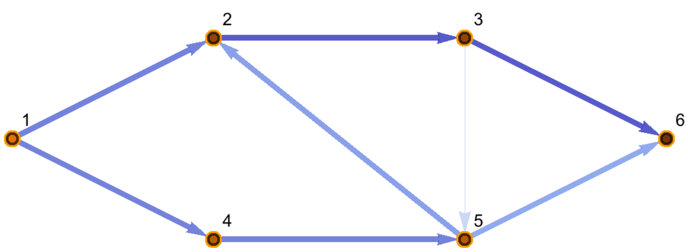
I call for an edge list.

```
g156["EdgeList"]
```

```
{1 ↦ 2, 1 ↦ 4, 2 ↦ 3, 3 ↦ 6, 4 ↦ 5, 5 ↦ 6, 5 ↦ 2}
```

and a flow graph.

```
g156["FlowGraph"]
```



I make a grid of the contributing edges and their contributions to the flow. The ghostly edge 3↦5 is not represented in the non-zero contribution list. From the grid, I see that I could either cut the graph to get the 14 flow units from 2↦3+5↦6 or from 1↦2+4↦5.

```
Grid[{#, g156[#]} & /@ g156["EdgeList"], Frame → All]
```

1 ↦ 2	7
1 ↦ 4	7
2 ↦ 3	11
3 ↦ 6	11
4 ↦ 5	7
5 ↦ 6	3
5 ↦ 2	4

I let Mathematica choose the cut. Mathematica chooses to get the 14 flow from 1↦2 + 4↦ 5.

```
FindMinimumCut[
 Graph[{1 ⟷ 2, 2 ⟷ 3, 3 ⟷ 6, 4 ⟷ 5, 5 ⟷ 6, 1 ⟷ 4, 5 ⟷ 2, 3 ⟷ 5},
  VertexLabels → "Name", VertexCoordinates ->
   {{-2, -1}, {0, 0}, {2.5, 0}, {4.5, -1}, {0, -2}, {2.5, -2}},
  EdgeCapacity → {20, 11, 13, 7, 3, 10, 4, 5}]]
{0, {{2, 3, 6, 5}, {1, 4}}}
```